BALANCE OF OPPOSING POTENTIALS WITH INFINITE TERMS IN THE SCHRÖDINGER EQUATION

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ABSTRACT. We consider the one-dimensional nonlinear Schrödinger equation,$iu_t + u_{xx} + \mathcal{N}(u) = 0, \quad x, t \in \mathbb{R}$, with the nonlinearity term that is expressed as a sum of powers, possibly infinite: $\mathcal{N}(u) = \sum c_n |u|^{\alpha_n} u, \quad \alpha_n > 0$. The combined nonlinearities appear in various physical applications such as chemical super fluidity, or the description of elementary particles such as bosons and defectons, or other subatomic structures, and in approximations of anisotropic media. We first investigate the local well-posedness of this equation for any positive powers of α in a certain weighted class of initial data, subset of $H^1(\mathbb{R})$. Then, using the pseudo-conformal transformation, we extend the local result to the global well-posedness. Furthermore, we investigate the asymptotic behavior of global solutions, those that have initial data with a quadratic phase $e^{ib|x|^2}$ with sufficiently large positive b, in particular, we prove scattering of these solutions in $H^1(\mathbb{R})$. One of the advantages of considering an infinite sum in the nonlinear term is the investigation of an exponential nonlinearity $e^{\alpha|u|}u$ and its well-posedness in that case, the first such result. We also show numerical simulations in the focusing case for various cases of combined nonlinearities, including the exponential one.

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