# TOTAL $k$-DOMINATION IN CARTESIAN PRODUCT OF COMPLETE GRAPHS

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ABSTRACT. Let $G = (V, E)$ be a finite undirected graph. A set $S$ of vertices in $V$ is said to be total $k$-dominating if every vertex in $V$ is adjacent to at least $k$ vertices in $S$. The total $k$-domination number, $\gamma_{kt}(G)$, is the minimum cardinality of a total $k$-dominating set in $G$. In this work we study the total $k$-domination number of Cartesian product of two complete graphs which is a lower bound of the total $k$-domination number of Cartesian product of two graphs. We obtain new lower and upper bounds for the total $k$-domination number of Cartesian product of two complete graphs. Some asymptotic behaviors are obtained as a consequence of the bounds we found. In particular, $\liminf_{n\to\infty}\left\{\frac{\gamma_{kt}(G\Box H)}{n} : G, H \text{ are graphs of order } n\right\} \leq 2\left(\left\lceil\frac{k}{2}\right\rceil^{-1}+\left\lfloor\frac{k+4}{2}\right\rfloor^{-1}\right)^{-1}$. We also prove that the equality is attained if $k$ is even. The equality holds when $G, H$ are both isomorphic to the complete graph, $K_n$, with $n$ vertices. Furthermore, we obtain closed formulas for the total 2-domination number of Cartesian product of two complete graphs of whatever order. Besides, we prove that, for $k = 3$, the inequality above is improvable to $\liminf\limits_{n\to\infty}\gamma_{3t}(K_n\Box K_n)/n \leq 11/5$.

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