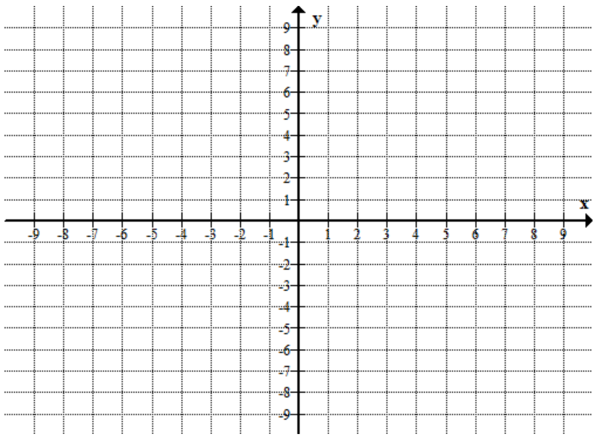
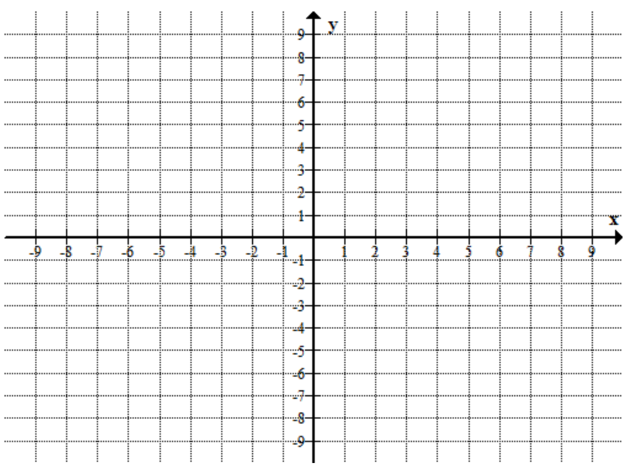
Review- Chapter 4

1. Use transformations to graph $f(x) = 3^{-x} + 2$. Determine the domain, range and find the equation of the horizontal asymptote. Plot at least 3 points on the graph of the basic function and **use them** to perform transformations. Do one transformation at a time and write the equation for each function.



2. The graph of $y = e^x$ is reflected about the y-axis and then shifted three units to the right. Write the equation of the function in the final position.

3. Use transformations to graph $f(x) = -2^{x+4} - 3$. Determine the domain, range and find the equation of the horizontal asymptote. Plot at least 3 points on the graph of the basic function and use them to perform transformations. Do one transformation at a time and write the equation for each function.



4. Express as a single logarithm. Assume all variables are positive and $b \neq 1$

   a) $2\log_b t - \frac{4}{5}\log_b s + \frac{1}{3}\log_b v - 4\log_b u$

   b) $2\ln x - 3\ln y - 4\ln z$

   c) $\log\left(\frac{p}{q}\right) + \log\left(\frac{q}{s}\right) - \log\left(\frac{p}{s^2}\right)$

5. Approximate $\log_5 (7)$ to 3 decimal places. (use the change of the base formula)

6. Use the properties of logarithms to find the exact values of the expressions. Do not use a calculator.

   a) $\log_4 24 - \log_4 6$

   b) $10^{\log 30 - \log 5}$

   c) $\log_3 30 \cdot \log_{30} 9$

7. Solve the given equation. Give exact solutions. Do not use a calculator.

a) $2^{x^2-3} = 64$

b) $e^{x-2} = \left(\frac{1}{e^2}\right)^{x-1}$

c) $e^{2x-1} = 2$

d) log(3+ x) – log(x-5) = log5

e) $\log_2(3x-2) - \log_2(x-5) = 4$

f) $3^x = 9^{x-1} \cdot 27^{1-3x}$

g) $\log(x^2 + 5x + 16) = 1$

h) $3 \cdot 2^{2x-1} + 5 = 14$

8. Let $f(x) = 2^x$. Describe a sequence of transformations that results in $g(x) = 3 \cdot 2^{2x-1} + 4$

9. Find the exact value of the logarithmic expression. Do not use a calculator.

a) $\log_3\left(\frac{1}{27}\right)$

b) $\log_5 \frac{1}{\sqrt{5}}$

10. Find the domain of

a) $f(x) = \log\left(\frac{x+2}{3-x}\right)$

b) $f(x) = \log_2\left(7x - x^3\right)$

c) $f(x) = \log\left(x^4 + x^3 - x^2 + x - 2\right)$

11. Suppose that ln 2 = a and ln5 = b. use properties of logarithms to write the given logarithm in terms of a and b.

a) ln 20

b) ln 2.5

c) $\ln \sqrt[3]{5}$

12. Solve the equation. Give exact values, do not use a calculator.

$$2^{1-x} = \left(\frac{3}{5}\right)^x$$

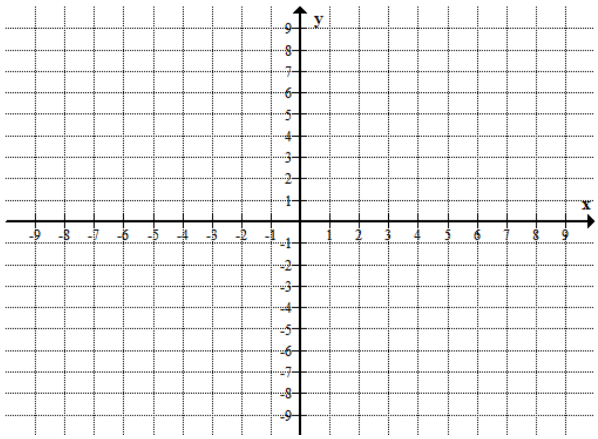
13. Change the logarithmic expression to an equivalent exponential expression

a) $\ln \frac{1}{e^5} = -5$

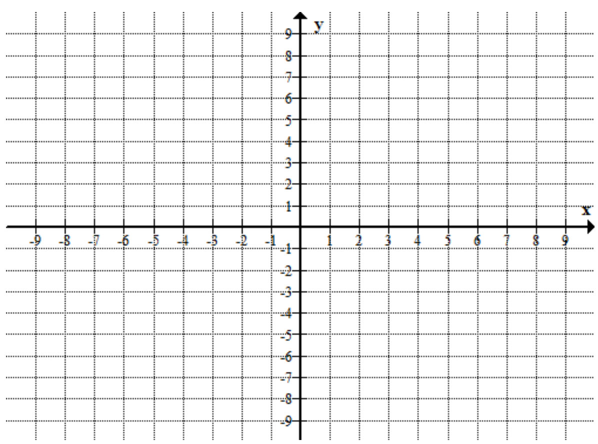
b) $\log_b 8 = 3$

14. Graph the given function using transformations. Plot at least 3 points on the graph of the basic function and use them to perform transformations. Do one transformation at a time and write the equation for each function.

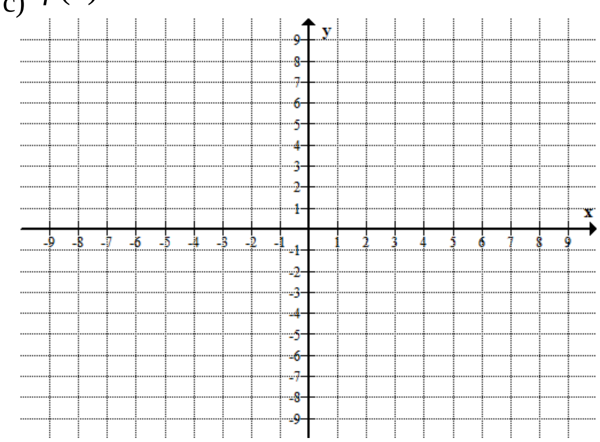
a) $f(x) = \log_2(-x+2)$



b) $f(x) = 2 - \ln(x+4)$



c) $f(x) = e^{\frac{1}{3}x} - 1$



15. Write as the sum and/or difference of logarithms. Express powers as factors. Assume x >9.

a) $\ln\left(\dfrac{7x\sqrt[9]{1+6x}}{(x-9)^7}\right)$

b) $\log_3\left(\frac{(3x-1)^2}{x^2\sqrt{x-9}}\right)$

16. Change the exponential expression to equivalent logarithmic expression $5^A = 4$

17. Solve logarithmic equations. Give the exact answer. Do not use a calculator. (Don't forget to consider the domain!)
   a) $\log_3 x^2 = \log_3(6x+7)$
   b) $\ln(x-6)+\ln(x+1)=\ln(x-15)$

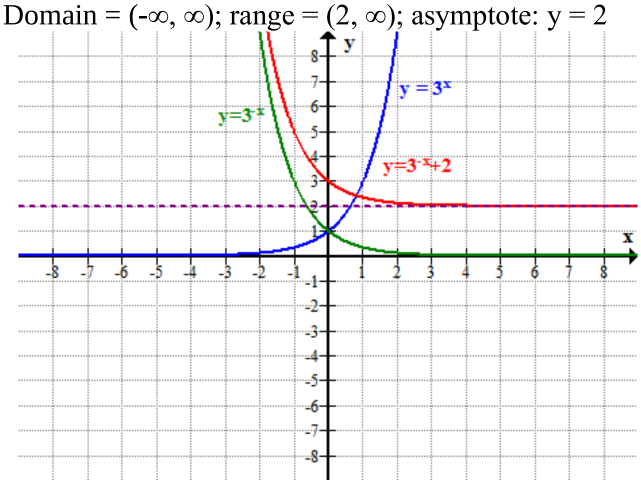
18. Let $f(x)=\log_2(x-2)$ and $g(x)=\log_2(4x+16)$. Solve the equation f(x) + g(x) = 6.

19. Solve the equation. Give exact values. Do not use a calculator
   $\log_6(x^2-x) = 1$

20. Let $f(x) = 2^x + 2^{-x}$ and $g(x) = 2^x - 2^{-x}$. Find $[f(x)]^2 - [g(x)]^2$.

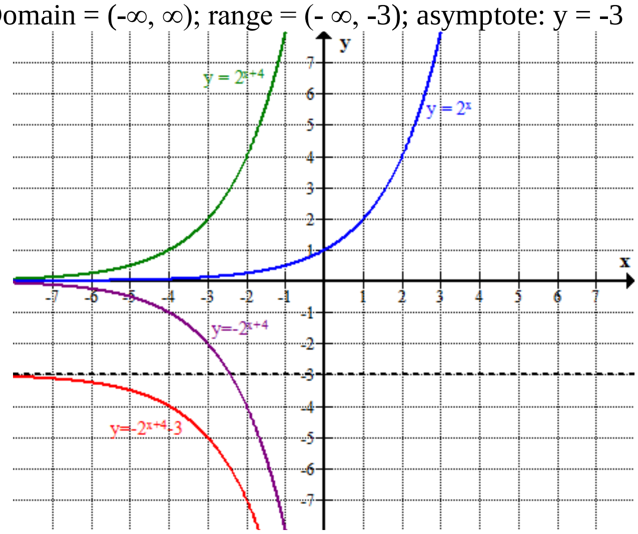
Answers:

1) Domain = (-∞, ∞); range = (2, ∞); asymptote: y = 2



2) $f(x) = e^{-x+3}$

3) Domain = (-∞, ∞); range = (- ∞, -3); asymptote: y = -3



4) a) $\log_b \dfrac{t^2 \cdot \sqrt[3]{v}}{\sqrt[5]{s^4} \cdot u^4}$; b) $\ln \dfrac{x^2}{y^3 z^4}$;c) log(s)

5) 1.209

6) a) 1; b)6; c) 2

7) a) -3,3; b) 4/3; c) $\dfrac{1+\ln 2}{2}$; d) 7; e) 6; f) 1/8; g) -3,-2; h) $\dfrac{\log_2(3)-1}{2}$

8) Shift to the right by 1; horizontal compression by a factor of 2; vertical stretch by a factor of 3; shift up by 4(other orders are possible)

9) a) -3, b) -1/2

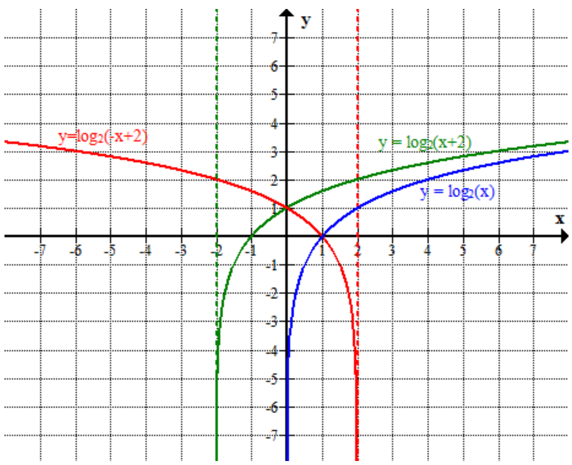
10) a) (-2,3) ; b) $(-\infty,-\sqrt{7}) \cup (0,\sqrt{7})$, c) $(-\infty,-2) \cup (1,+\infty)$

11) a) 2a+b; b) b-a; c) b/3

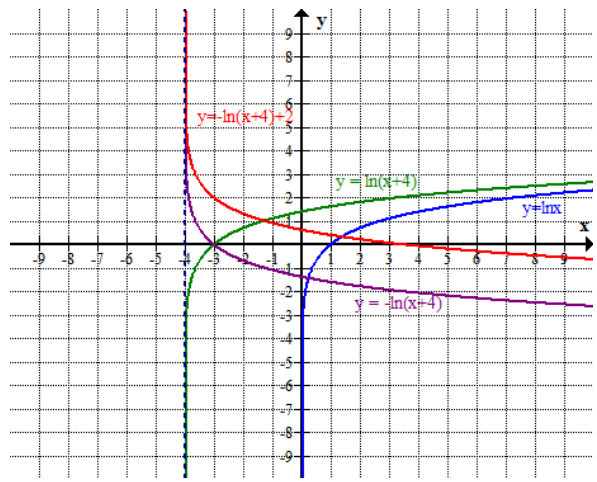
12) $\dfrac{\ln 2}{\ln(6/5)}$

13) a) $\dfrac{1}{e^5} = e^{-5}$; b) $b^3 = 8$

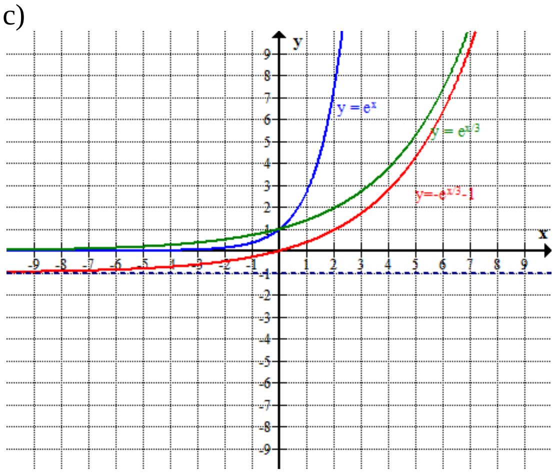
14) a)



b)



c)



15) a)ln7+ lnx + (1/9)ln(1+6x) – 7ln(x-9); b) $2\log_3(3x-1) - 2\log_3 x - \frac{1}{2}\log_3(x-9)$

16) A= $\log_5(4)$

17) a) -1, 7; b) no solution

18) 4

19) -2, 3

20) 4